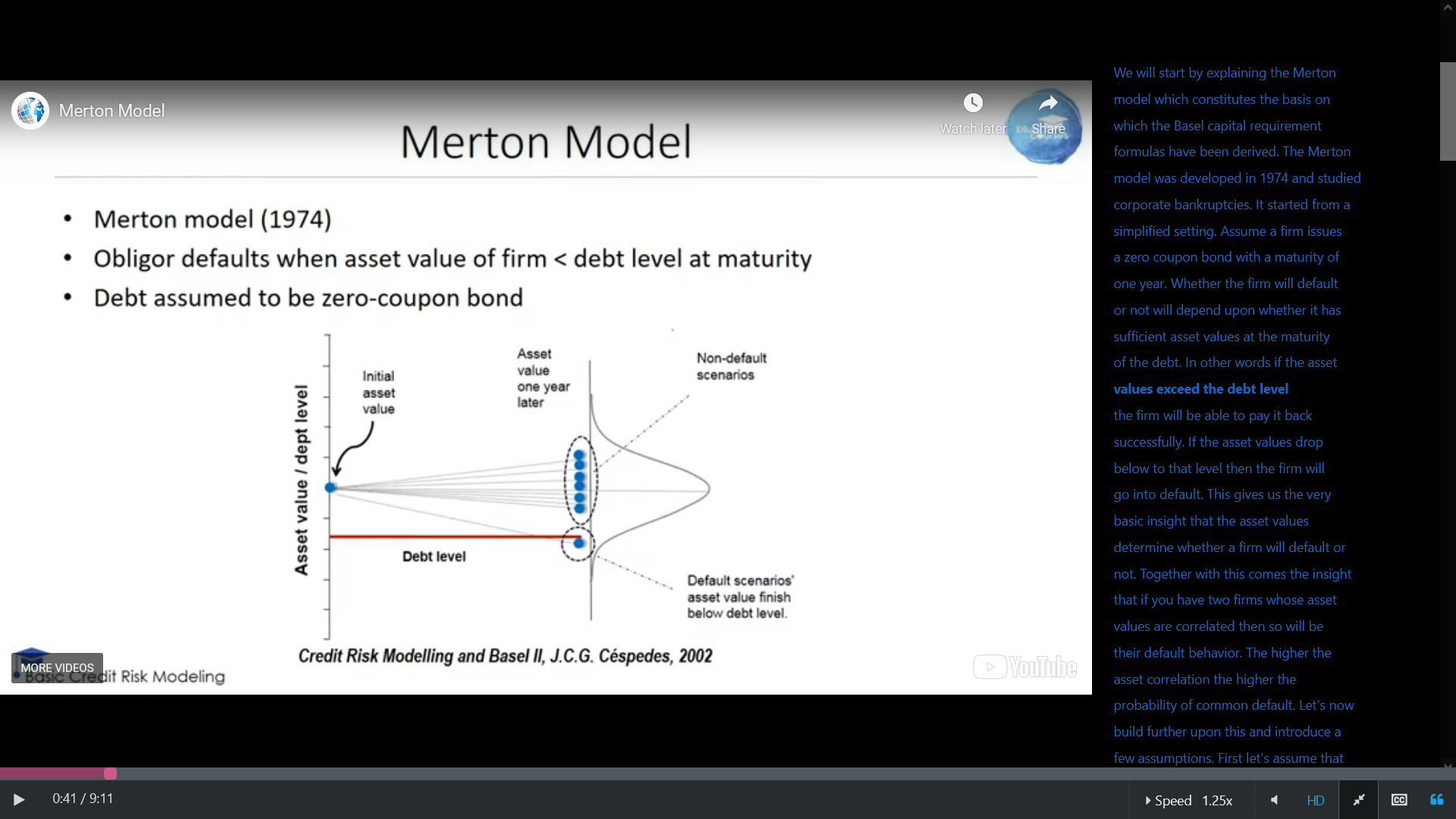
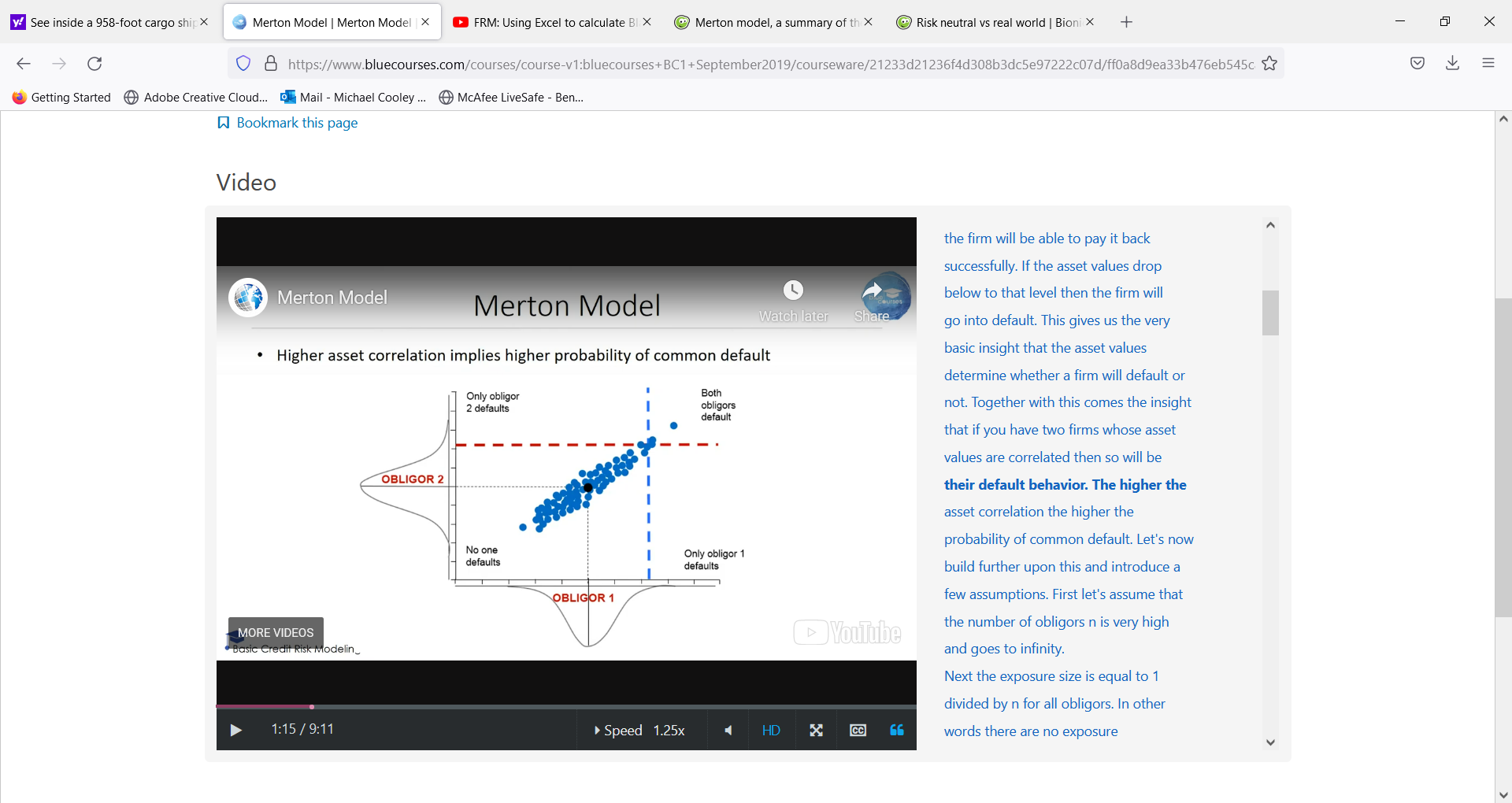
We will start by explaining the Merton model which constitutes the basis on which the Basel capital requirement formulas have been derived. The Merton model was developed in 1974 and studied corporate bankruptcies. It started from a simplified setting.

Assume a firm issues a zero coupon bond with a maturity of one year. Whether the firm will default or not will depend upon whether it has sufficient asset values at the maturity of the debt. In other words, if the asset values exceed the debt level the firm will be able to pay it back successfully. If the asset values drop below to that level, then the firm will go into default. This gives us the very basic insight that the asset values determine whether a firm will default or not.



Together with this comes the insight that if you have two firms whose asset values are correlated then so will be their default behavior. The higher the asset correlation the higher the probability of common default.



Let's now build further upon this and introduce a few assumptions.

1. First let's assume that the number of obligors n is very high and goes to infinity.
2. Next the exposure size is equal to 1 divided by n for all obligors. In other words, there are no exposure concentrations.
3. We will also assume that all obligors have the same probability of default. This makes sense when the Merton model is applied at the level of a credit rating.
4. Since each credit rating has its own PD we also assume equal pairwise asset correlation rho between any two of obligors.

Vasicek now formulated the model as follows: the assets of firm i are equal to the square root of rho times the factor f which is an economic factor common to all companies plus the square root of 1 minus rho times an idiosyncratic shock epsilon\_i which represents the company specific risk.

Let's now assume that both f and epsilon\_i are mutually independent standard normally distributed variables. Hence the assets of firm i also follow a standard normal distribution.

It can be easily shown that the correlation between the asset values of firm i and j works out to rho and is thus independent of the firm. The unconditional probability of default PD is then the probability that the asset values of firm i are less than or equal to the debt level Di. Since the asset values follow a standard normal distribution this can be calculated by using the cumulative standard normal distribution evaluated at Di here written down as N(Di).

The conditional probability of default given the systemic factor f PD\_i given f is equal to the probability that Ai is less than or equal than D\_i given f. This works out as the probability of the square root of 1 minus rho times epsilon\_i being less than or equal to D\_i minus the square root of rho times f given f. This is equal to the probability of epsilon\_i being less than or equal to the ratio of D\_i minus the square root of rho times f and the square root of 1 minus rho.

Because epsilon\_i follows a standard normal distribution the conditional probability of default given the systematic factor f can be calculated by evaluating the cumulative standard normal distribution at D\_i minus the square root of rho times f divided by the square root of 1 minus rho. From our earlier unconditional probability, we have D\_i equal to the inverse cumulative standard normal distribution evaluated at PD\_i. We also put the systematic factor f representing the state of the economy to its 99.9% worst level which is equal to the inverse cumulative standard normal distribution evaluated at 0.001, the latter being equal to minus the inverse cumulative standard normal distribution evaluated at 0.999.

This gives us the conditional probability of default equal to the cumulative standard normal distribution evaluated for the sum of the inverse cumulative standard normal distribution evaluated at PD\_i and the square root of rho times the inverse cumulative standard normal distribution evaluated at 0.999 divided by the square root of 1 minus rho. We can also refer to this conditional probability as alpha star.

Now what does this alpha star represent? Actually, you can interpret it as the 99.9 percent worse case default rate based upon the average PD and the asset correlation rho. So, you can think of it as a conservative PD estimate when you look at the loss distribution as depicted here. The alpha star is the situated far right in the tail. Note that this is also often referred to as a Value at Risk or VaR calculation. The Basel Accord will now use this alpha star to calculate its capital. Remember alpha star is the loss such as the probability of observing a low or equal than loss is 99.9% meaning there is a 0.1 percent chance or once in 1000 years that a banks capital would fail to absorb the unexpected loss and become insolvent.

The total loss is further decomposed into the expected loss which is the PD and the unexpected loss which is the difference between alpha star and the PD. The expected loss should be covered by provisions. Provisions are part of the profit and loss statement which are set aside for expected losses. The Basel capital Accord then focuses on the unexpected loss which is the difference between alpha star and the PD. This brings us to the Basel capital requirements formula. here you can see the Basel capital requirements formula depicted for retail exposures. The formula boils down to alpha star minus PD, which represents the unexpected loss remember, multiplied by the severity of the loss or LGD. By multiplying the obtained number K with the exposure at default the regulatory capital is obtained.

One critical number in this formula is the asset correlation rho. Remember the Merton model was introduced in a corporate setting where it is rather easy to calculate the asset correlation. However for retail exposures this is less straightforward. That's why the asset correlation factors have been written down in the Accord.

* For residential mortgages the asset correlation has been set to 0.15,
* for qualifying revolving exposures to 0.4 and
* for other retail exposures it depends on the PD as you can see depicted. The higher the PD the lower the asset correlation.

A similar formula has been introduced for corporate, sovereigns and central bank exposures as you can see right here. Note that it also includes a maturity adjustment term and a b factor which have both been set based upon empirical analysis. Here also the asset correlation rho depends upon the PD. The maturity M represents the nominal or effective maturity and is between one and five years. For exposures to large financial sector entities rho is multiplied by 1.25. Also, a firm size adjustment for small and medium-sized enterprises has been included based upon the total annual sales in millions of euros.